Lecture 6: 02/06/2012

Particle Acceleration:

It is important to understand how particles in high-energy sources accelerate to Lorentz factors required to emit X-ray and γ-ray photons. Although larg structures like molecular clouds or galaxy clusters can contribute significantly to the ambient high-energy intensity, the overwhelming majority of energetic particles are produced in and around compact objects (such as white dwarfs, neutron stars, and black holes).

Particle acceleration in and around compact objects is due to the strong fields induced by these objects. Here we discuss acceleration of particles in gravitational and electromagnetic fields. The main purpose will be to compare the optimal efficiency of various processes in transferring energy from the fields to the particles.

Gravitational Field:

A particle falling from infinity onto an object of mass M and radius R can acquire a maximum velocity:

$$v_{max} = \left(\frac{2GM}{R}\right)^{\frac{1}{2}}$$

For a white dwarf we have:

$$v_{max}(wd) \simeq \left(\frac{2GM_\odot}{R_{wd}}\right)^{\frac{1}{2}} \simeq 7.4 \times 10^{8}\ \frac{cm}{s} \Rightarrow \gamma_{max}(wd) \simeq 1.0003$$

It is clear that gravitational field of a white dwarf alone cannot produce relativistic particles.

Next consider a neutron star:

$$v_{max}(ns) \simeq \left(\frac{2GM_\odot}{R_{ns}}\right)^{\frac{1}{2}} \simeq \frac{c}{2} \Rightarrow \gamma_{max}(ns) \simeq 1.19$$

This is definitely more interesting than a white dwarf.

However, $\gamma_{max}(ns)$ is still far too small for a particle like electron or proton to radiate energetic γ-ray photons.

For example, to radiate a 30 GeV γ-ray photon we need $\gamma \simeq 30$ (for a proton) and $\gamma \simeq 60,000$ (for an electron).

It is also worth noting that photon produced near a compact object undergo gravitational redshift. As we discussed last time the redshift factor is $\simeq \frac{GM}{Rc^2}$. For objects like a white dwarf or neutron star the increase in γ is much larger than the redshift of the emitted photon. However, for a black hole the gravitational redshift becomes infinite as the particle gets very close to the event horizon. As a result, the increase in the redshift more than offsets the increase in γ. Therefore very little high energy emission is expected from regions near the event horizon.

Electromagnetic Field:

It is clear from our discussion that nongravitational acceleration schemes must play a role in energizing the particles in many high-energy sources. The most common is electromagnetic acceleration, via at least two methods of energy transfer. Depending on the field distribution, various mechanisms may contribute to the acceleration. When the magnetic field $\vec{B}$ is turbulent or random, the principal method is the Fermi acceleration. In this process, disordered bundles of magnetic flux act as mirrors to bounce particles back and forth, increasing their energy with every collision. We will discuss this in detail later on. For a well-organized field, a more direct acceleration mechanism is based on the idea that a component of the electric field $\vec{E}$ may be generated parallel to $\vec{B}$, where the particle motion is unrestricted. This is the situation we consider first.

To understand how a magnetic field disturbance energizes the charges, let us go through some of the basic ideas of magnetohydrodynamics (MHD). First, consider Maxwell's equations (in Gaussian units):

$$\vec{\nabla} \cdot \vec{E} = 4\pi \rho_e$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$$

Here ρ_e is the density of electric charge and $\vec{J}$ is the current density. Additional relations are found by considering a highly conducting medium such as a plasma of a fully ionized gas. This is a typical situation in the magnetosphere of a pulsar. A pulsar can be considered as a rotating magnetized sphere (thus a rotating magnetic dipole). The induced electric field by this rotation ionizes the gas in the magnetosphere. In the presence of $\vec{B}$ and $\vec{E}$ fields in a highly conducting medium, we have:

$$\vec{E} + \frac{\vec{v}}{c} \times \vec{B} = 0$$

Here $\vec{v}$ is the velocity field of the plasma. Moreover, the conservation of mass leads to the following equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

ρ is the density of the plasma (not to be confused with ρ_e).

The equivalent of the Newton's second law for a continuous medium is:

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho(\vec{v}\cdot\vec{\nabla})\vec{v} = \rho_e \vec{E} + \frac{1}{c}(\vec{J}\times\vec{B})$$

Here we have used the fact that $\frac{d}{dt} \equiv \frac{\partial}{\partial t} + (\vec{v}\cdot\vec{\nabla})$. For an electrically neutral medium $\rho_e = 0$. After using the Maxwell's equations, we find:

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho(\vec{v}\cdot\vec{\nabla})\vec{v} \simeq -\frac{1}{4\pi}\vec{B}\times(\vec{\nabla}\times\vec{B})$$

Together with the conservation of mass and the condition for a highly conducting medium:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

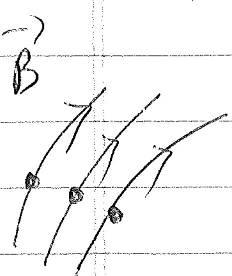
$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}\times\vec{B})$$

These three equations form a complete set of coupled equations.

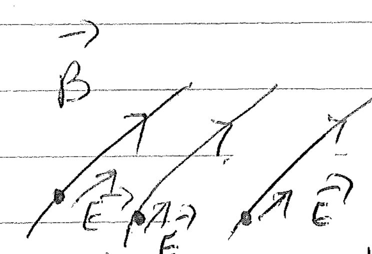
Suppose now that the field $\vec{B}$ is subject to a disturbance. In a pulsar, this can happen for example as a result of a crustal disturbance. The solution to the above equations that describes a propagating plane wave is:

$$\begin{cases} \vec{B} = B_0 \hat{z} + B_A \exp(ikz - i\omega t)\,\hat{x} \\ \vec{v} = v_A \exp(ikz - i\omega t)\,\hat{x} \\ \vec{E} = E_A \exp(ikz - i\omega t)\,\hat{y} \end{cases}$$

This is called the Alfven wave (hence subscript "A"). We notice the presence of an electric field. However, it is perpendicular to $\vec{B}$. Pulsar have very strong magnetic fields (up to 10^{12} G). For such strong magnetic field, charged particles can only move along the magnetic field lines. Acceleration along these lines requires a component of the $\vec{E}$ field that is parallel to $\vec{B}$:



Charges attached to $\vec{B}$ lines
(no parallel component of $\vec{E}$)



Charges accelerated along $\vec{B}$ lines
($\vec{E}$ has a parallel component)

A parallel component can arise if the plane wave has a finite extension. In reality, one expects some structure in the x-y plane (due to the crust). A solution that is permitted by the above equation and takes into account of this is:

$$\vec{B} = B_0 \hat{z} + B_A \sin(k_y y) \exp(ikz - i\omega t) \hat{x}$$

$$\vec{E} = \frac{ic}{\omega} [ik B_A \sin(k_y y) \hat{y} - k_y B_A \cos(k_y y) \hat{z}] \exp(ikz - i\omega t)$$

The z-component of $\vec{E}$ can now result in acceleration of charged particles. The equation of motion for an electron along the z direction is:

$$\frac{dP_z}{dt} = eE_z \Rightarrow \frac{d}{dt}(\gamma m_e v_z) = eE_z$$

For relativistic motion $v_z \simeq c$. The major contribution to the left hand side therefore comes from $\frac{d\gamma}{dt}$. We find:

$$\frac{d\gamma}{dt} \simeq \frac{eE_z}{m_e c}$$

This equation is valid between two successive collisions of the electron with particles in the plasma. Thus:

$$\gamma_{max} \simeq \frac{eE_z}{m_e c \nu_c}$$

Here ν_c is the collision frequency. In a typical pulsar magnetosphere, where the particle density is $\sim 10^{16} - 10^{26}\ cm^{-3}$, we have $\nu_c \sim 10^{17}\ s^{-1}$. Also, in typical pulsars, $k_y \sim \frac{2\pi}{\lambda_{crust}}$ where the crust scale length $\lambda_{crust} \sim 10\ cm$. In addition, $\omega \simeq v_\alpha k$, where v_α is the phase velocity of the Alfven wave and $k \sim \frac{2\pi}{R_{ns}}$. We find that an electron can be accelerated to a Lorentz factor of 10^{10} or more. In practice, several damping factors (such as pair creation) will set in before reaching this value.